P. Nason

$\alpha_{\rm em}$, $\alpha_{\rm em}(M_{\rm Z})$, $(g-2)_\mu$

$\alpha_{\rm em}$ is measured with great precision at very small momentum. In physical processes, when the momentum transfer is not small $\alpha_{\rm em}$ becomes bigger. This is a consequence of the screening property of electromagnetism: at a distance Δx from a charge, all virtual pairs of particles of mass greater than $1/\Delta x$ screen the charge. Thus

$\alpha_{\rm em}({\rm Thomson})$: totally screened

$\alpha_{\rm em}(s)$ we must compute (or measure) the screening effect of all charges of mass less than $\sqrt{s}$.

The screening effect of strong interacting particles cannot be computted. A sum rule relates the value of $R(e^+e^-\to{\rm hadrons})$ to the value of α.

Very high precision experiments (EW tests at LEP, $(g-2)_\mu$) drive the need to measure $R(e^+e^-\to{\rm hadrons})$ with very high accuracy.

$\alpha_{\mathrm{em}}(M_{\mathrm{Z}})$

At LEP, $\sin^2\theta_{\mathrm{W}}$ is determined most precisely from

$$\sin^2\theta_{\mathrm{W}}(1-\sin^2\theta_{\mathrm{W}}) = \frac{\pi\alpha_{\mathrm{em}}(M_{\mathrm{Z}})}{\sqrt{2}G_{\mathrm{F}}M_{\mathrm{Z}}^2}$$

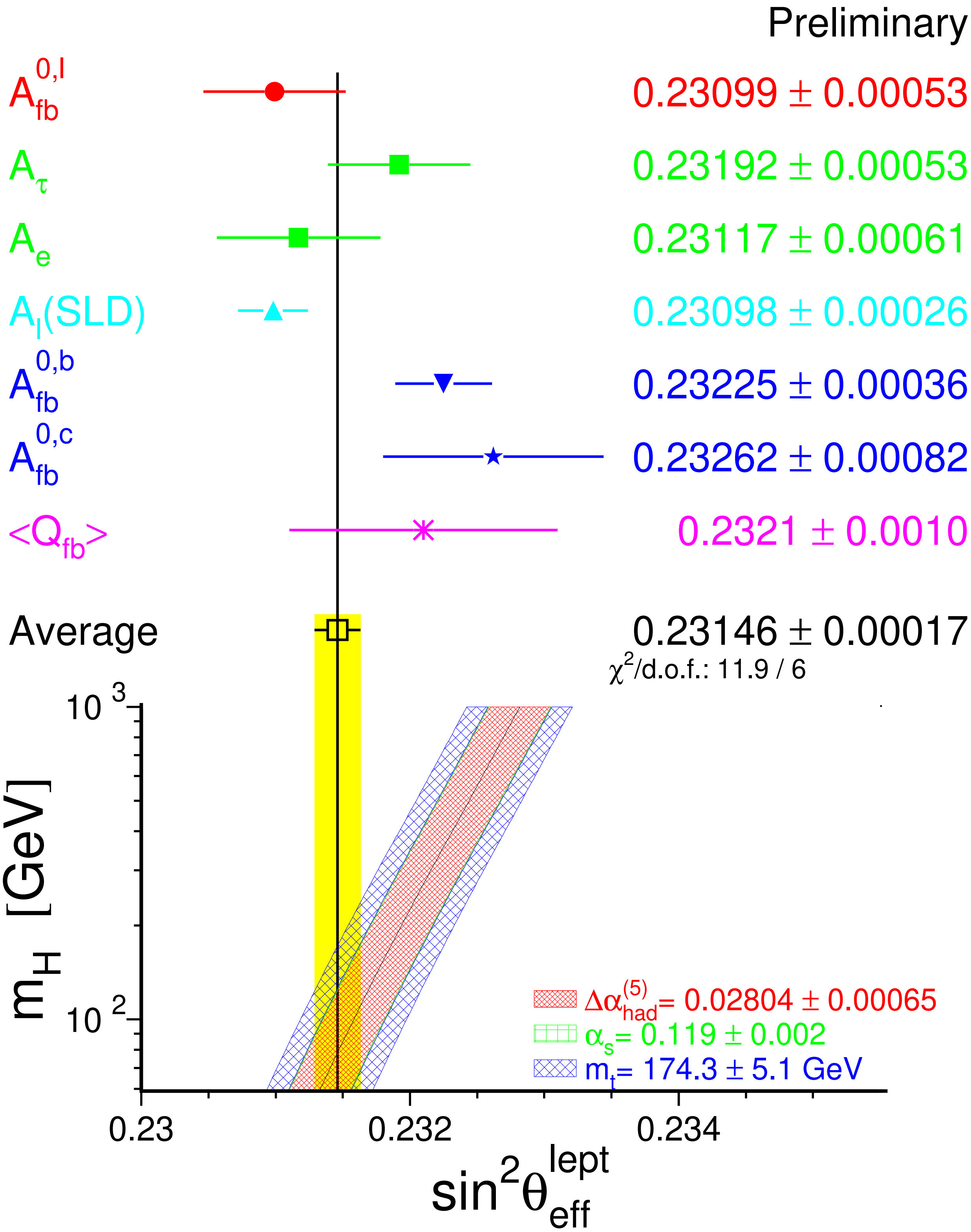
with M_{Z} known to 23 ppm, G_{F} to 9 ppm, and $\alpha_{\mathrm{em}}(M_{\mathrm{Z}})$ to 600 ppm. The impact of these errors on $\sin^2\theta_{\mathrm{W}}$ is rougly

$$\begin{aligned}
\frac{\delta\sin^2\theta_{\mathrm{W}}}{\sin^2\theta_{\mathrm{W}}} &= 1.4\frac{\delta\alpha_{\mathrm{em}}}{\alpha_{\mathrm{em}}} \\
&= 1.4\frac{\delta G_{\mathrm{F}}}{G_{\mathrm{F}}} \\
&= 2.8\frac{\delta M_{\mathrm{Z}}}{M_{\mathrm{Z}}}
\end{aligned}$$

where the dominant error is coming from $\alpha_{\mathrm{em}}(M_{\mathrm{Z}})$. The above definition of $\sin^2\theta_{\mathrm{W}}$ equals other definitions at the tree level (it differs from them by M_{t} and M_{H} dependent radiative corrections). The $\alpha_{\mathrm{em}}(M_{\mathrm{Z}})$ error propagates directly to other definitions.

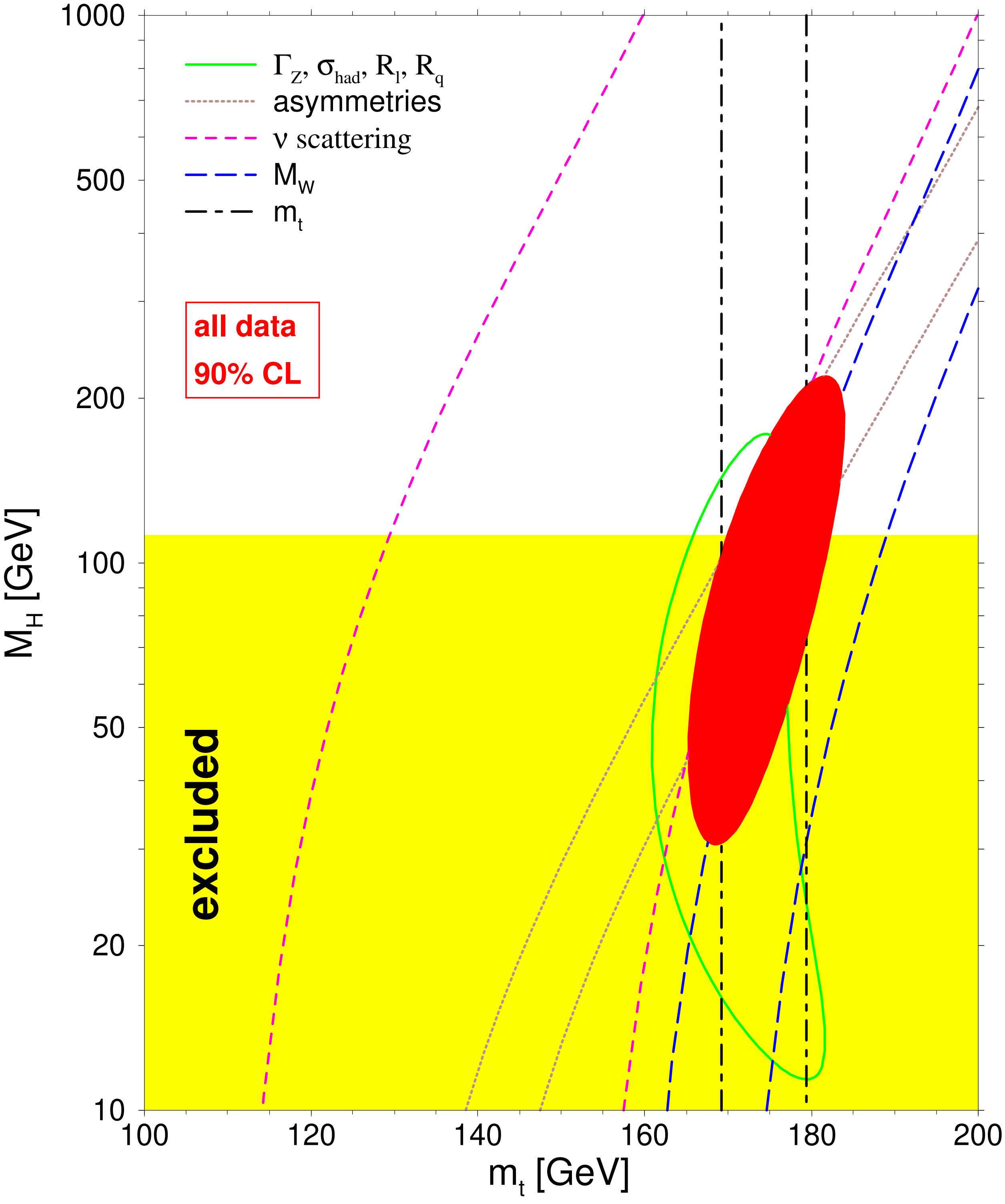
This is the case for the asymmetry defined $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$.

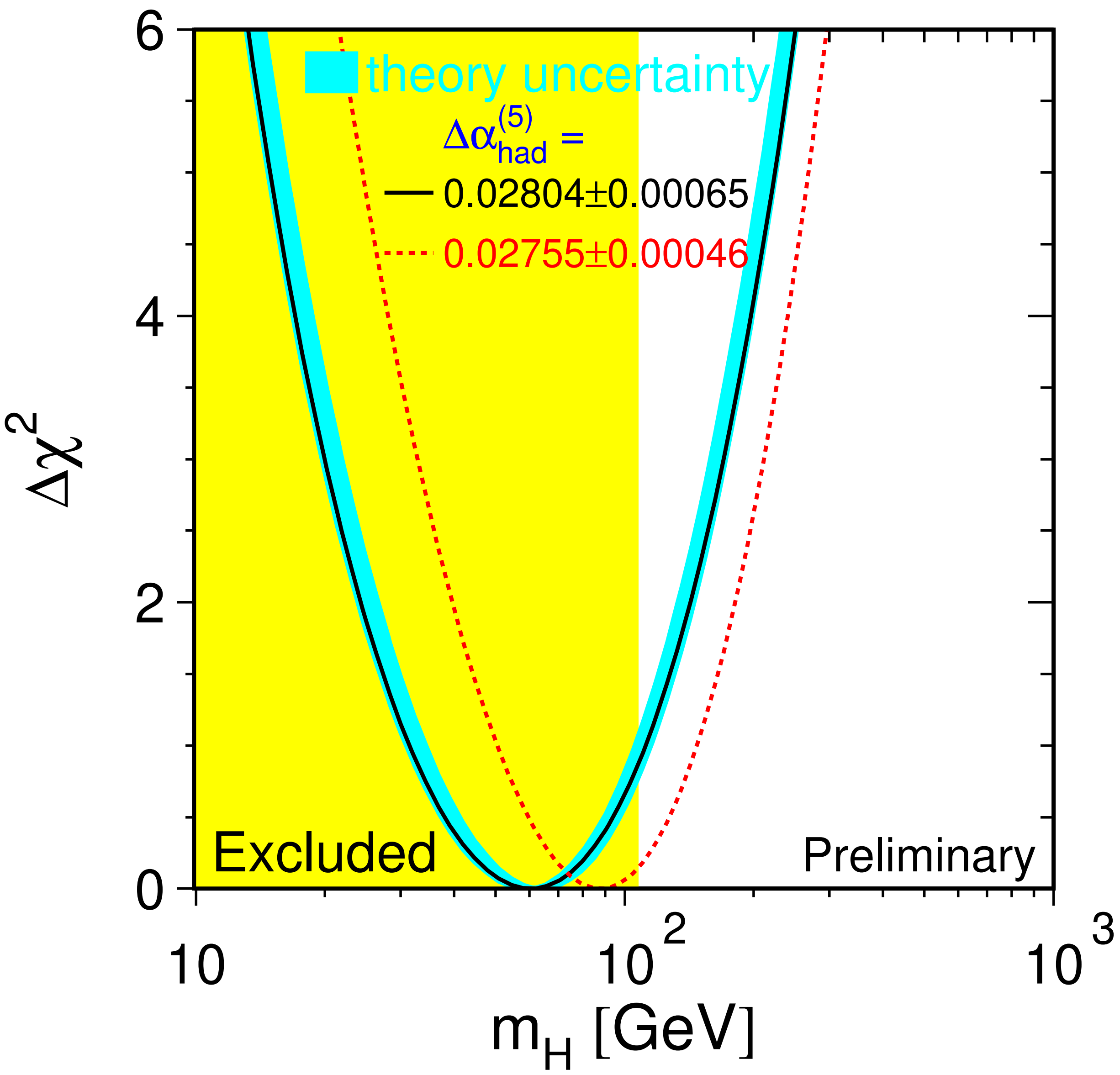
From the LEPEWWG, for example



the red band amount to $2 \times 650\,10^{-6} \times 1.4 \times 0.23$.

Asimmetries strongly constrain the $M_{\rm t}$, $M_{\rm H}$ plane, thus the large error from $\Delta\alpha_{\rm em}$ has a strong impact on the limits on the Higgs mass.





theory uncertainty
$\Delta\alpha^{(5)}_{had}$ =
0.02804±0.00065
0.02755±0.00046
Excluded
Preliminary
$\Delta\chi^2$
m_H [GeV]
6
4
2
0
10
10^2
10^3

Given the short distance (fundamental) value of $\alpha_{\rm em}(M_{\rm Z})$, polarization effects alter its value at long distances. Since $\alpha_{\rm em}(M_{\rm Z})$ is better mesured at infinite distances, all polarization phenomena from $M_{\rm Z}$ down to the electron mass have to be removed in order to obtain $\alpha_{\rm em}(M_{\rm Z})$.

$$\alpha_{\rm em}(s) = \frac{\alpha_{\rm em}}{1 - \Delta\alpha(s)}$$

While leptonic polarization effects can be computed, the hadronic part is obtained from a dispersion integral of the R_γ value

$$\Delta\alpha^{\rm hadr} = -\frac{\alpha_{\rm em}}{3\pi}\mathcal{P}\int_{4m_\pi^2}^{\infty} ds' R_\gamma(s') \left(\frac{1}{s'-s} - \frac{1}{s'}\right)$$

with

$$R_\gamma(s) = \frac{\sigma(e^+e^- \to \gamma^* \to {\rm hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)}$$

In the dispersion integral, one can use data for small scales, and perturbative QCD for large scales.

Eidelman and Jegerlehner (1995) have used existing e^+e^- data for $R\gamma$ up to $E_c \approx 40$ GeV, and pQCD for $s' > E_c^2$. They obtain

$$\Delta\alpha_{\rm had}(M_{\rm z}) = 0.02804 \pm 0.00064$$

The error (2.3%) comes mostly from experimental errors on $R\gamma$ measurements. The parton model value of the PQCD contribution is

$$\begin{aligned}\Delta\alpha^{\rm hadr}_{\rm pert} &= -\frac{\alpha_{\rm em}}{3\pi} N_c \left(2\frac{4}{9} + 3\frac{1}{9}\right) \int_{E_c^2}^{\infty} ds' \left(\frac{1}{s'-s} - \frac{1}{s'}\right) \\ &= 0.0028 \log\frac{S - E_c^2}{E_c^2} = 0.004\end{aligned}$$

Assuming that the QCD correction to the parton model value is of the order of 5% and its uncertainty is 0.5%, this correction can be easily estimated with an error of 0.00002. Their estimate would not change much if we did not believe at all the pQCD correction to the parton model formula.

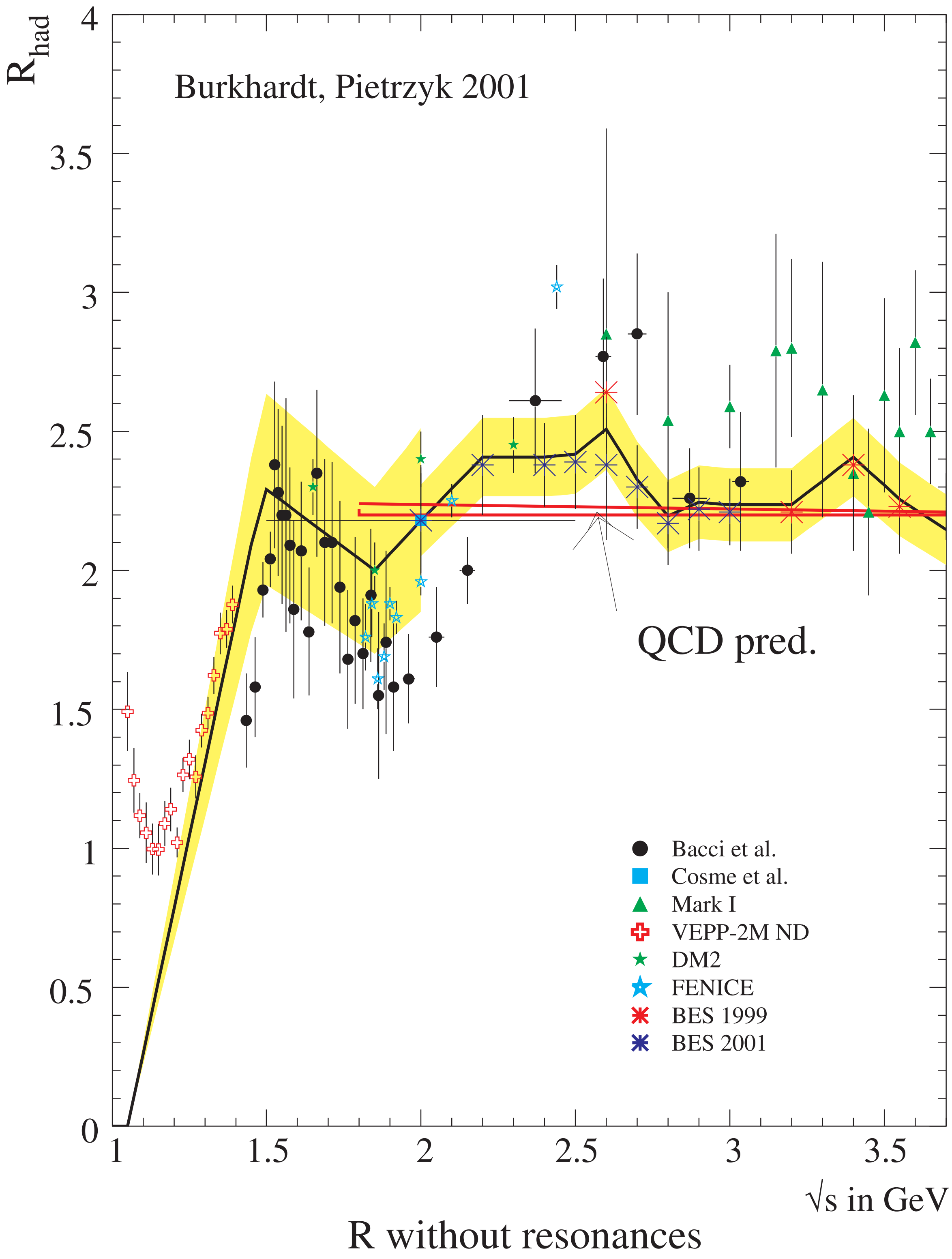
If we could push the E_c scale down to 1.8 GeV, we would have a parton model contribution of 0.022, a pQCD correction of (roughly) 0.0022, and an error on it of the order of 0.00022.

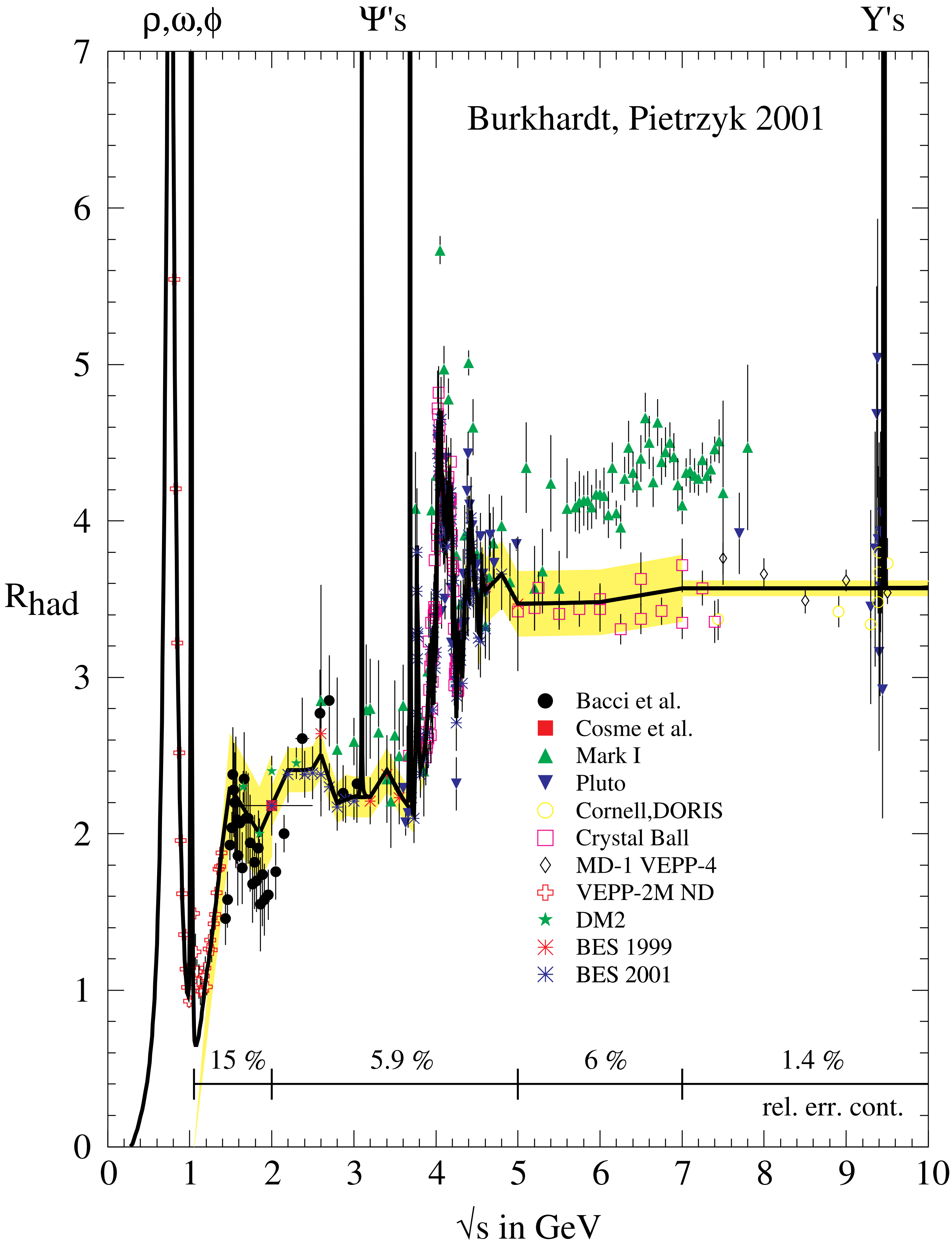
2 avenues to improve on the error

- Trust more the theory
- Get more data

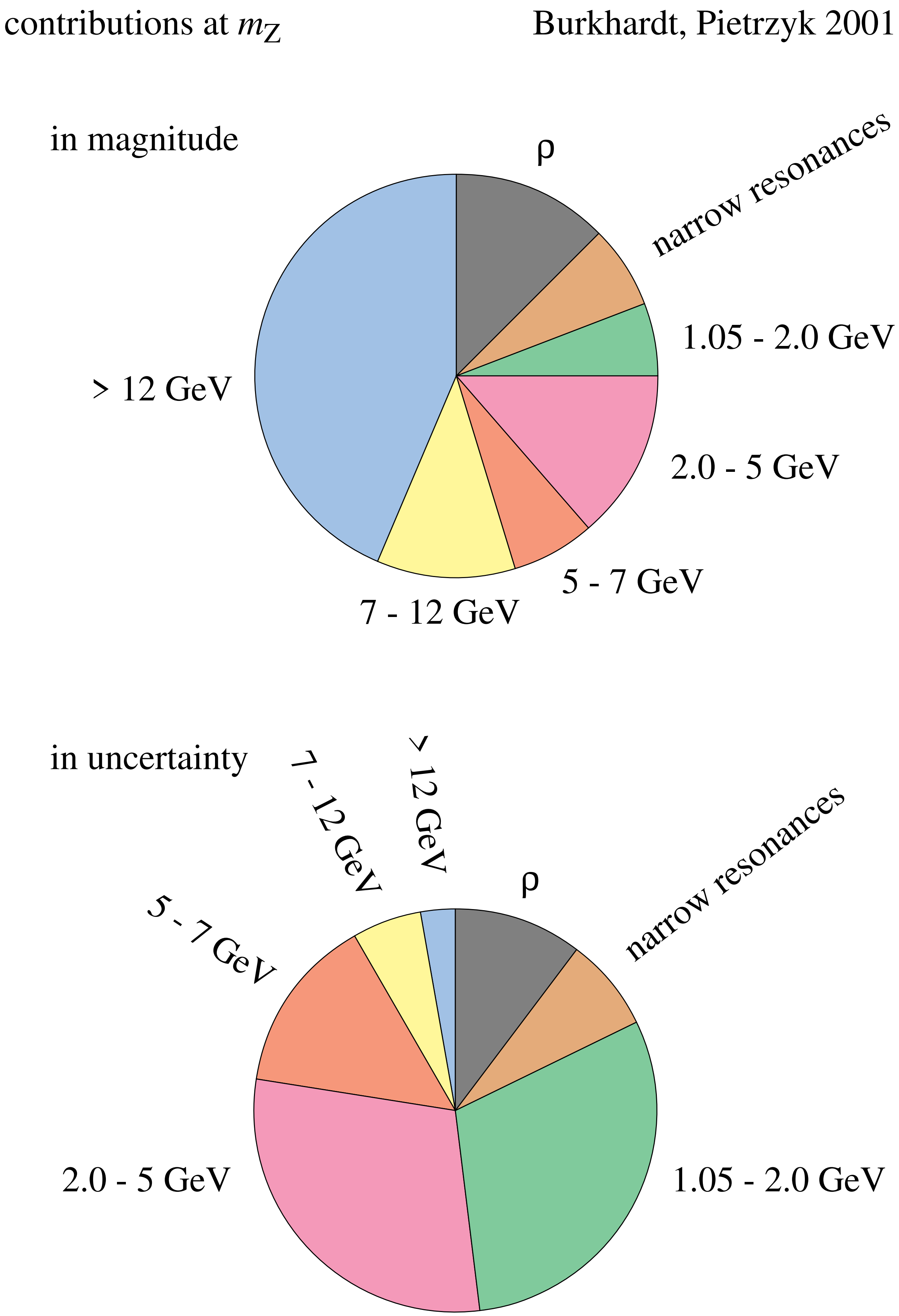
Recent BES measurements have been included in the analysis of Burkhardt and Pietrzyk (in earlier analysis, without BES data, they gave a result identical to that of Eidelman and Jegerlehner). In 2001 they give

$$\Delta\alpha_{\rm had}(M_{\rm z}) = 0.02761 \pm 0.00036$$

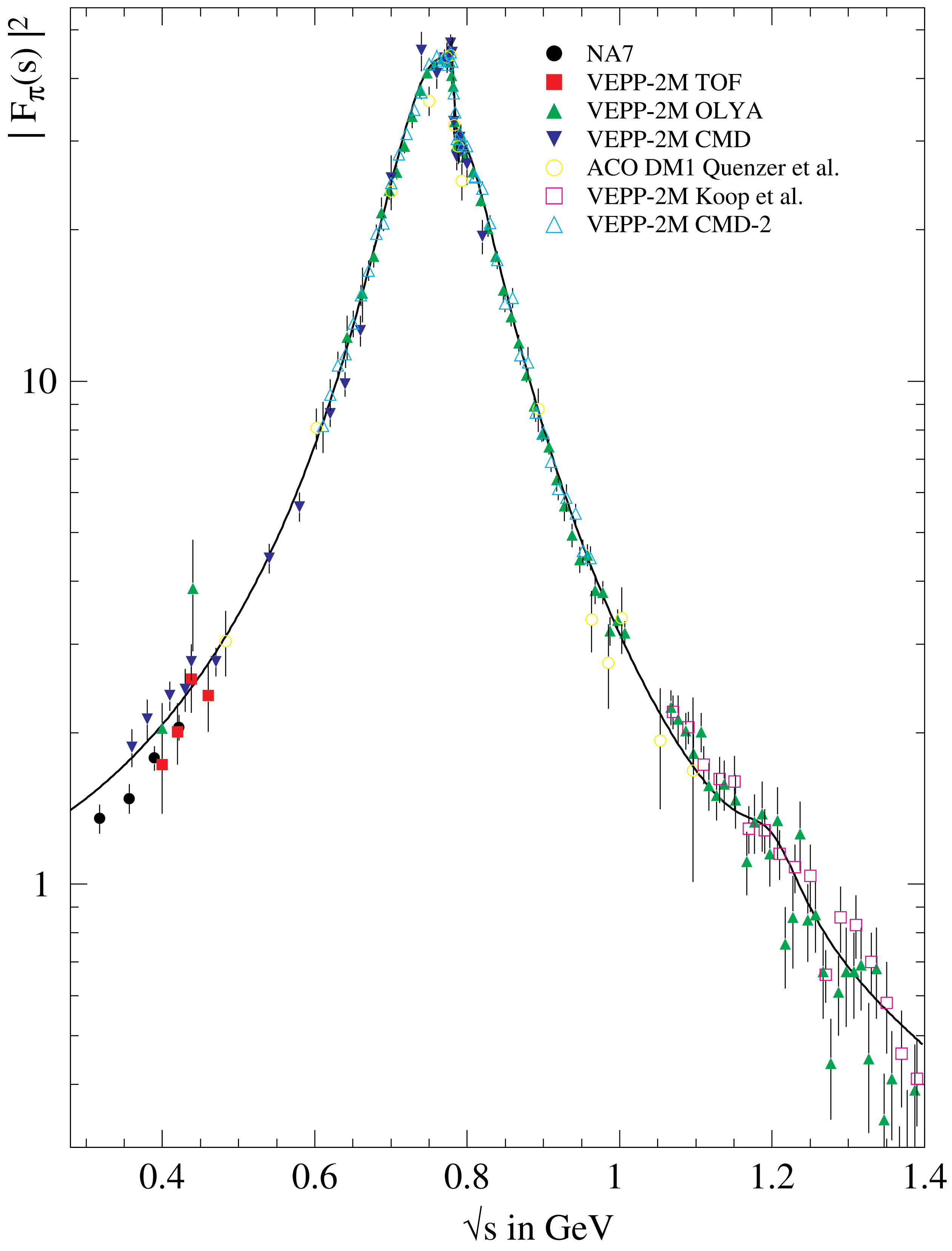




ρ,ω,φ
Ψ's
Υ's
Burkhardt, Pietrzyk 2001
Rhad
Bacci et al.
Cosme et al.
Mark I
Pluto
Cornell,DORIS
Crystal Ball
MD-1 VEPP-4
VEPP-2M ND
DM2
BES 1999
BES 2001
15 %
5.9 %
6 %
1.4 %
rel. err. cont.
√s in GeV
0
1
2
3
4
5
6
7
8
9
10



contributions at m_Z
Burkhardt, Pietrzyk 2001
in magnitude
ρ
narrow resonances
1.05 - 2.0 GeV
2.0 - 5 GeV
5 - 7 GeV
7 - 12 GeV
> 12 GeV
in uncertainty
7 - 12 GeV
> 12 GeV
ρ
narrow resonances
5 - 7 GeV
2.0 - 5 GeV
1.05 - 2.0 GeV



$|F_\pi(s)|^2$
10
1
0.4
0.6
0.8
1
1.2
1.4
$\sqrt{s}$ in GeV
NA7
VEPP-2M TOF
VEPP-2M OLYA
VEPP-2M CMD
ACO DM1 Quenzer et al.
VEPP-2M Koop et al.
VEPP-2M CMD-2

Few remarks on theory driven approaches

From our formula

$$\Delta\alpha^{\mathrm{hadr}} = -\frac{\alpha_{\mathrm{em}}}{3\pi}\mathcal{P}\int_{4m_\pi^2}^{\infty} ds' R_\gamma(s') \left(\frac{1}{s'-s} - \frac{1}{s'}\right)$$

The function

$$I(s) = \int_{4m_\pi^2}^{\infty} ds' R_\gamma(s') \frac{1}{s'-s}$$

is an analytic function for complex s, except for the positive real axis. For large values of $|s|$, it can be computed in pQCD up to a constant (well, an infinite constant), as a power expansion in $\alpha_{\mathrm{S}}(s)$. The presence of the subtraction term at $s = 0$ in the formula for $\Delta\alpha_{\mathrm{em}}$ prevents the possibility to compute it from pQCD alone. It is clear, however, that something can be computed:

- R_γ itself at large values of s. $R_\gamma(s)$ is the imaginary part of $I(s)$. It works only away from resonance regions.
  - Davier, Höker (98): 0.02778 ± 0.00026
  - Kühn, Steinhauser (98): 0.02777 ± 0.00017 (also correct charm data normalization comparing with pQCD just above and below the charm resonance region
- Polinomial finite energy sum rules on R_γ. The integral of

$$\oint_{|s|=s_0} I(s)P(s)ds \propto \int_0^{s_0} R_\gamma(s)P(s)ds$$

Davier, Höker (98): 0.02763 ± 0.00016 (use FESR to suppress problematic regions in the data)

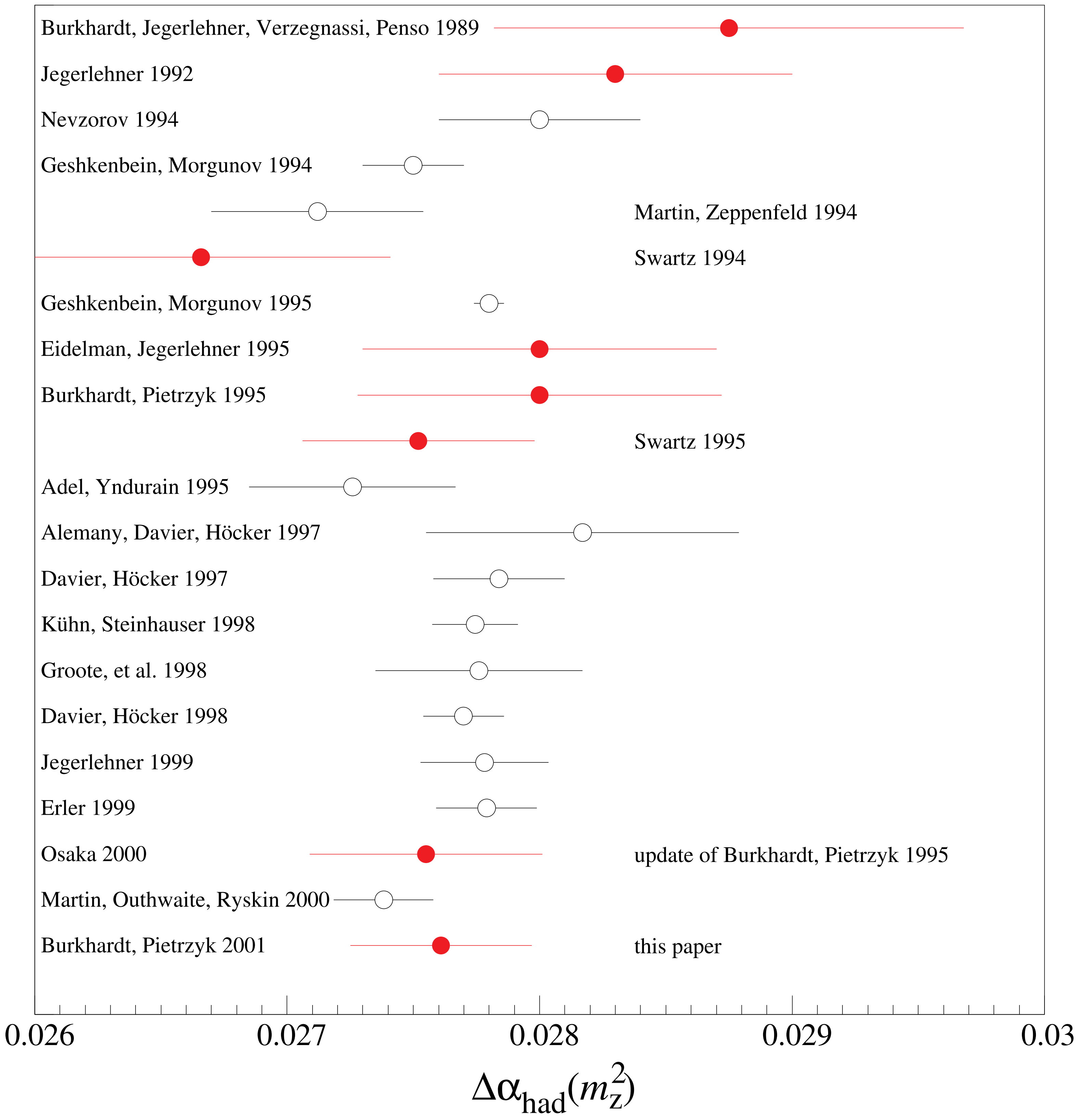
- $I(s)$ for negative s is not sensitive to resonances; one can write

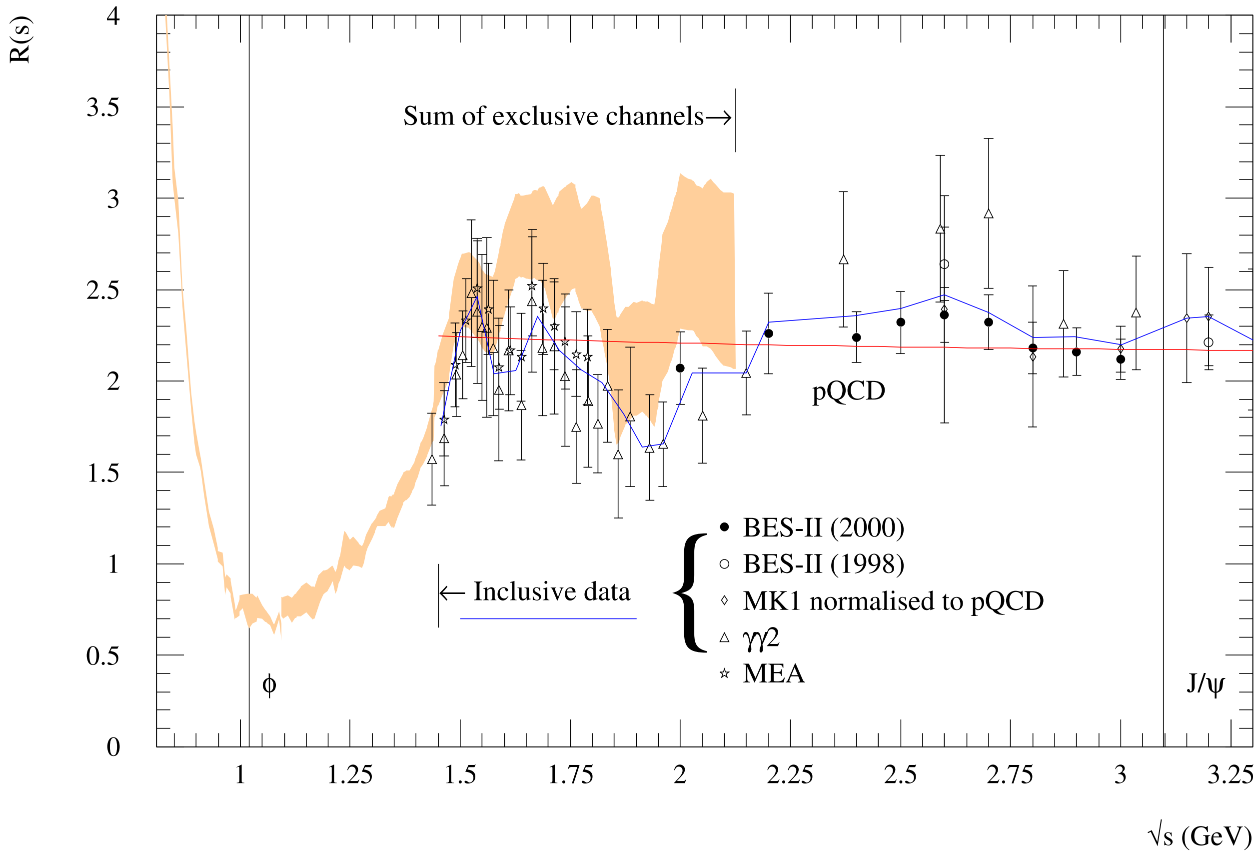
$$\Delta\alpha_{\rm em}^{\rm hadr}(-M_{\rm Z}^2) \propto (I(-M_{\rm Z}^2) - I(-s_0)) + (I(-s_0) - I(0))$$

evaluate the first term with pQCD, and the second term with data (it is less sensitive to higher energy data). Jegerlehner (1999): 0.02778 ± 0.00024

- Other analiticity tricks
  Martin, Outhwaite, Ryskin(2000): 0.02741 ± 0.00019 (include 2000 BES data)

Summary from Burkhardt and Pietrzyk paper





Martin etal emphasise the discrepancy, in the range $1.5 < \sqrt{S} < 2.1$, between different sets of data. BES data seems to favour the inclusive data, which at the end they use, and get

- $\Delta\alpha_{\rm had}(M_{\rm z}) = 0.02743 \pm 0.00019$ using inclusive data in the ambiguous region
- $\Delta\alpha_{\rm had}(M_{\rm z}) = 0.02765 \pm 0.00021$ using exclusive data in the ambiguous region

It is clear without further work that there is room of improvement in the range $1.5 < \sqrt{S} < 2.1$.

$(g-2)_\mu$

Some notation: one usually defines

$$a_\mu = \frac{g-2}{2}$$

The first computation of a_μ is by Schwinger

$$a_\mu = \frac{\alpha_{\rm em}}{2\pi} = 0.0011617....$$

Today, corrections up to the fifth order have been computed. The precision of the QED part of the calculation is enormous. Taking $\alpha_{\rm em}$ from the $(g-2)_e$ one gets

$$a_\mu(\mathrm{QED}) = 11658470.57(0.29) \times 10^{-10}$$

weak corrections are

$$a_\mu(\mathrm{weak}) = 15.2(0.4) \times 10^{-10} \ .$$

The BNL result is

$$a_\mu = 11659202(14)(6) \times 10^{-10}$$

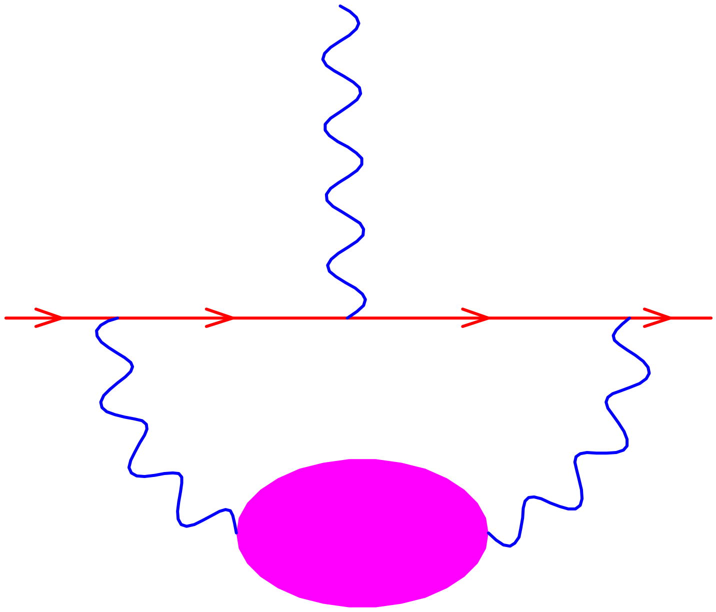
with a claimed deviation from the SM value

$$a_\mu - a_\mu(\mathrm{SM}) = 43(16) \times 10^{-10} \ .$$

with the promise to bring the error down to 4×10^{-10}.

Hadronic corrections

Hadronic corrections enter at $\mathcal{O}(\alpha_{\rm em}^2)$, through the graph



which gives a contribution of about 600×10^{-10}. It cannot be calculated. One must reconstruct the vacuum polarization from available R measurements.

The total hadronic contribution is given by

$$a_\mu^{\rm had} = a_\mu^{\rm had}({\rm vac.pol.1}) + a_\mu^{\rm had}({\rm vac.pol.2}) + a_\mu^{\rm had}({\rm lightbylight})$$

where ${\rm vac.pol.2}$ amounts to (roughly) -10×10^{-10}, and represents the $\mathcal{O}(\alpha_{\rm em}^3)$ corrections including vacuum polarization contributions. The evaluation of this term does not pose problems, since it is not required with high accuracy.

We have

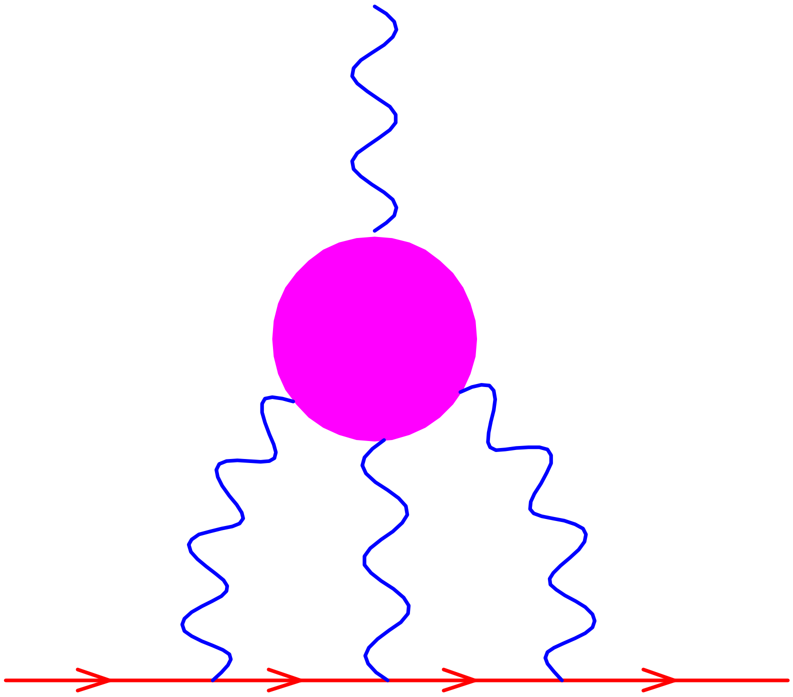
$$a_\mu^{\rm had}({\rm vac.pol.1}) = \left(\frac{\alpha_{\rm em}}{3\pi}\right)^2 m_\mu^2 \int_{4m_\pi^2}^{\infty} \frac{ds}{s^2}\, K(s/m_\mu^2)\, R(s)$$

where K is a bounded function, approaching 1 at infinity. So, the integrand goes roughly like $1/s^2$ in this case, compared to $1/(s(s-M_{\rm Z}))$ in the case of $\alpha_{\rm em}(M_{\rm Z})$. The low energy region is much more important. The situation is summarized in the following table (roughly)

range	$a_\mu^{\rm had}({\rm vac.pol.1})$	error
$\sqrt{s} < 1.4$ GeV	599×10^{-10}	$11.6(3.4) \times 10^{-10}$
$1.5 < \sqrt{s} < 2.5$ GeV	44×10^{-10}	4.6×10^{-10}
$\sqrt{s} > 2.5$ GeV	55×10^{-10}	$4.6(2.3) \times 10^{-10}$

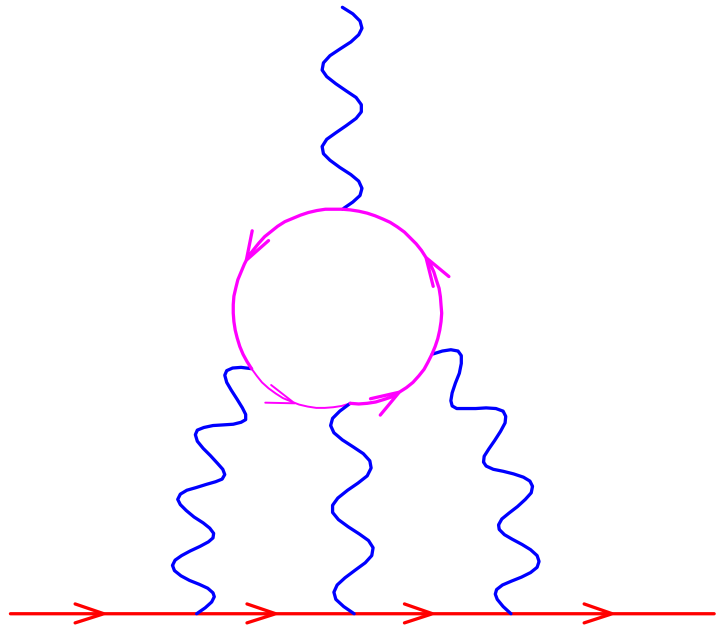
the region from $1.5 < \sqrt{s} < 2.5$ needs improvement to meet the promised 4×10^{-10} accuracy.

The light by light contribution is problematic, since it cannot be measured



currently quoted estimates are around -9×10^{-10}. They are obtained in a calculation in which strong interactions are fudged with a theory of massive constituent quarks, integacting via four fermion interactions, with a UV cutoff. All calculations normally quoted belong to this framework, which, however has no solid theoretical justification.

Worse of all, Ettore Remiddi has estimated the light by light contribution by replacing the red blob by a loop of a massive quark (160 MeV).



The mass value is such that with the same trick the polarization blob comes out just about right. Remiddi finds a positive light by light contribution, 15×10^{-10}, instead of the -8.5×10^{-10} used in present analysis. The BNL discrepancy becomes

$$a_\mu - a_\mu(\mathrm{SM}) = 19.5(16) \times 10^{-10}$$

instead of $43(16) \times 10^{-10}$.

The contribution computed in this model remains positive for all values of the quark masses. Even gluon exchange in the quark loop give positive contribution. Of course, as Ettore himself would say, this is just a model estimate. But so is the other...

A better way to estimate the light light contribution is badly needed before any statement can be made out of the BNL result